

National Textile Center

FY 2003 (Year 12) Project Proposal

Project No.

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Competency: Management Systems

An Analytical Investigation of the Bullwhip Effect

Project Team:

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Objective:

The bullwhip effect is a well-established, frequent, and expensive occurrence: modest fluctuations in consumer demand are dramatically amplified as they proceed up the supply chain from retailer to apparel manufacturer to textile manufacturer. For the first time, exact, analytical solutions to the supply chain equations are available, and we propose to use these new solutions to develop truly optimal ordering policies that will reduce inventory fluctuations. The goals of the research are 1) To advance the mathematical understanding of the new solutions, both theoretically and through numerical integrations. 2) To broaden the set of solutions to cover the wide range of textile and apparel inventory problems. 3) To produce practical heuristics and guidelines for reducing the Bullwhip fluctuations, and generally improving inventory management. The overall objective is the definition of cost-effective ordering policies that result in a stable and robust response of the inventory to variations in customer demand.

Relevance to NTC Mission:

Inventory management is a critical issue throughout the textile and apparel industry. Sharp spikes and drops in demand result in stock-outs and inventory excesses. The mismatch between forecast and demand is amplified as orders are transmitted along the supply chain. This "Bullwhip Effect" severely impacts both profitability and customer satisfaction. Therefore, there are significant benefits to Bullwhip reduction in particular, and from improved inventory management in general. Previous efforts have relied on approximations and simulations, but the new analytical solutions should result in much better managerial understanding of the impacts of real world policies such as price promotions, order batching, manufacturing delays, etc.

State of the Art:

Inventory control remains an important and thorny problem. Burbidge's Law states "if a demand is transmitted along a series of inventories using stock control ordering, then the amplitude of demand variation will increase with each transfer." Burbidge pioneered the simulation approach, and modern simulation approximations have led to further refinements (e.g., Sourcing Simulator, NTC I01-S01, I01-S10). Recently, Lee popularized the problem using the term "Bullwhip Effect," [1] which has been documented as a significant problem for a wide variety of textile and apparel companies. While numerous theoretical techniques have been employed, all involved approximations. e.g., Lee [1] assumed the demand was serially correlated in a linear programming, cost minimization model. Control theory techniques provide global stability theorems, but do not describe the detailed inventory behavior over time.[2]

It appears that Warburton [3] was the first to solve analytically the entire set of time-delayed inventory and ordering equations. While that solution included a step function surge in user demand, the approach is quite general, and its application should provide a whole new approach to inventory control. Sterman's popular and practical management heuristics are directly related to the equations solved here. Therefore, our model covers a wide variety of realistic supply chain problems in textiles and apparel.

[1] H. L. Lee, V. Padmanabhan, S. Whang, Information distortion in the Supply Chain: The Bullwhip Effect, *Management Science*, 43, 4, (1997) 546-558.

[2] S.M. Disney, M.M. Naim, D.R. Towill, Genetic algorithm optimization of a class of inventory control systems. *Int. J. Production Economics*, 68 (2000) 259-278.

[3] R. D. H. Warburton, Exact analytical solutions for the production inventory control problem, Submitted to *Int. J. Production Economics* (2002).

Approach:

Warburton's solution to the supply chain equations was obtained using the Lambert W function, which was popularized through efficient and accurate implementations, such as in Maple. The rich and complex dynamical behavior of the inventory response can be traced directly to the properties of the Lambert W function. (See Figure 1) The solutions were validated in several ways. First, a direct numerical integration of the equations gives the same inventory response as the theoretical solutions. In particular, the theoretical solutions predict the transition from stable to unstable inventory behavior. This critical stability point was confirmed by very precise numerical integrations.

The proposed research extends these new solutions with three parallel tasks:

a) Extending the Mathematic

We will continue to explore the mathematical properties of the solutions. For example, the Final Value Theorem (FVT) applied in a control theory representation led to the discovery of permanent

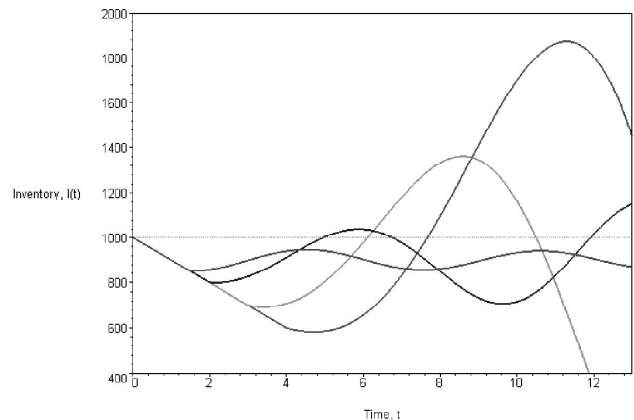


Figure 1. For small manufacturing delays, the inventory oscillates, but results in a permanent deficit. The oscillations grow dramatically as the delay time increases, eloquently suggesting a relation to the Bullwhip Effect.

inventory deficits.[2] These deficits were confirmed by examining the Lambert W based solutions as $t \rightarrow \infty$. However, the analytical solutions additionally predicted the size of the deficits. We intend to further investigate the correspondence with the control theory approach, which should provide significant insight into the global response of the inventory to realistic demands. Also, further comparisons of the exact analytical solutions with numerical integrations should provide guidelines for the numerical accuracy of simulations employed elsewhere. (I01-S10)

b) Calibration of the Bullwhip Effect

Lee suggested that there were 5 major causes to the Bullwhip Effect: users interpreting the orders (demand), long manufacturing times, batching of orders, price promotions (which artificially stimulate demand), and supply shortages. Since our exact solutions provide the inventory response over time, we can determine the optimal order rate that returns the inventory to a desired value without Bullwhip Effects. (See Figure 2.)

The orders can be followed as they progress up the supply chain, and the inventory response determined at each echelon. We will calculate the growth in orders from retailer to apparel manufacturer, and then from apparel manufacturer to textile supplier. Thus, we propose to characterize the Bullwhip Effect in terms of the real world, managerial parameters. To date, this has only been accomplished in approximate solutions, or for restrictive assumptions. The tuning of the parameters, coupled with the properties of the Lambert W function, should result in smaller Bullwhip Effects, and generally improved inventory management policies. Figure 2 shows that by varying the ordering policy, the inventory response can be controlled, and that it is possible to determine precisely the optimal policies that reduce the Bullwhip Effect.

c) Generalizing Order-up-to (OUT) Inventory Policies

OUT policies are known to be optimal when the demand is stationary. This is an unrealistic assumption since the demand is usually unknown and often highly variable. Hence, there is a mismatch between the forecast and actual demand, and the error is amplified as it is transferred up the supply chain. This demonstrates the importance of understanding the response of the inventory to different types of user demand. We propose to analyze a wide variety of consumer demands. For example, an oscillation in demand is interesting because it represents a situation that frequently occurs: promotions, and other artificial demand stimuli, result in a surge of orders followed by a drought. Our goal is to generalize beyond the OUT policies, and determine an optimal, generally applicable ordering policy that produces a stable and robust inventory response to any type of demand fluctuation. Apparel and textile manufacturers, will be able to plan their inventory management ahead of fashion cycles, promotions, and other business strategies, so as to minimize the typical short-term stock-outs and long term inventory excesses.

Mathematical Approach

Companies attempt to minimize inventory, while keeping sufficient on hand to protect against demand fluctuations. While we focus on the retailer's inventory, the approach is quite valid for any echelon of the supply chain. The retailer's inventory, $I(t)$, is depleted by consumer demand, $D(t)$, and increased by receipts, $R(t)$, from

manufacturers, equation (1). Receipts are equal to the orders, just delayed by the manufacturing time, τ , (equation 2) which results in a differential delay equation. The receipts depend on the ordering policy, which directs the actual inventory towards its desired value. The ordering policy, $O(t)$, is typically defined with three terms: the sum of the exponentially smoothed demand, with an average of T_a time units, plus a fraction, $1/T_i$, of the inventory deficit, plus a fraction, $1/T_w$, of the work in process (WIP) deficit, equation (4).

$$dI/dt = R(t) - D(t) \quad (1) \quad R(t) = O(t - \tau) \quad (2) \quad O_i(t) = [I_o - I(t)]/T_i \quad (3)$$

$$O(t) = d\{1 - \exp(-t/T_a)\} + O_i + O_w \quad (4) \quad O_w(t) = [WIP_o - WIP(t)]/T_w \quad (5)$$

Smoothing is typically applied to the demand data, otherwise excessive fluctuations occur, resulting in increased costs. Exponential smoothing is efficient and the method of choice, equation (4). The inventory deficit term (3) brings the actual inventory towards the desired inventory, and the constant, T_i , spreads the deficit recovery over time. It is imprudent to recover the entire deficit in one time period.[2] However, long delay times result in a significant Bullwhip Effect. The Work in Process (WIP) term (5) allows the ordering rate to depend on the orders in the pipeline. If a surge occurs, the WIP is depleted, and it is desirable to increase the order rate. On the other hand, excessive WIP should result in a decreased ordering rate. The three tunable parameters are T_a , T_i , and T_w .

It was only recently demonstrated [3] that these equations could be solved exactly in terms of the Lambert W function, opening up a whole new range of research opportunities. Figure 1 shows several such analytical solutions, which were validated through numerical integration. For small values of the delay time, τ , the solutions approach a constant value, which agrees with results obtained by control theory, as discussed above. The solutions oscillate and diverge for larger values of the delay time, eloquently suggesting a relation to the Bullwhip Effect. The model can be expressed in block diagram form, which is fundamental to the control theory approach. The same model has been used extensively in simulations.

Practically useful information can be deduced immediately: the peak in the inventory can be calculated by setting the derivative of the inventory = 0; and the period of oscillation is determined from the complex exponential part of the solution. Most importantly, one can compute exactly the combination of parameters for which the inventory response changes from stable to unstable. This is of vital practical interest since a diverging inventory results in huge excess costs.

Figure 2 shows an example of the power of the analytical approach. By varying the WIP recovery, the response of the inventory can be tuned to any desirable behavior.

Our model covers a wide variety of realistic textile and apparel supply chain problems. The precise behavior of the inventory can be carefully assessed over a wide range of conditions, which makes the model extremely useful in precisely defining and quantifying the bullwhip effect, elucidating the contribution of the various parameters, assessing the underlying causes, and measuring the relative impact of proposed remedies. Since the use of simulation without a sound underlying theoretical basis can be hazardous, the exact, analytical solutions presented here should also prove useful in validating several previous NTC projects. (I01-S10, I01-S01).

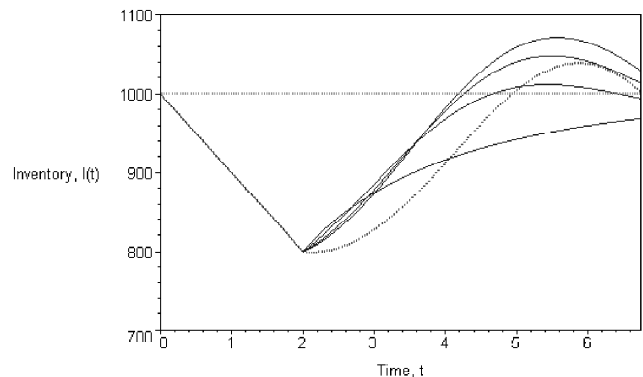


Figure 2: The inventory response can be controlled by varying the WIP parameter, T_w . For the dotted line, there is no WIP parameter.

This Year's Goal:

We will solve the equations for a series of consumer demand behaviors, and determine ordering policies that minimize inventory fluctuations along the supply chain. We will suggest ways that the optimal ordering policies can be incorporated into software systems, which would then become available to the industry as a whole.

Outreach to Industry:

Griffin manufacturing, a leader in quick response manufacturing, has agreed to provide real world supply chain data to validate our models, and to test the proposed management policies.

New Resources Required:

No new resources required.